Exercice 1 : Déterminer la forme canonique des fonctions trinomes suivantes :

1. $f(x) = -2x^2 + 12x - 14$
2. $f(x) = 2x^2 - x + 1$
3. $f(x) = 2x^2 - x - 1$
4. $f(x) = 2x^2 - x - 15$
5. $f(x) = \frac{1}{2}x^2 - x - \frac{3}{2}$
6. $f(x) = -\frac{1}{5}x^2 - 2x - 5$
7. $f(x) = -\frac{1}{3}x^2 + x + 3$
8. $f(x) = 3x^2 - x + \frac{5}{12}$

Exercice 2 : Résoudre les équations suivantes :

1. $4x^2 + 12x + 9 = 0$
2. $x^2 - \sqrt{2}x + \frac{1}{2} = 0$
3. $3x^2 - \sqrt{6}x + 1 = 0$
4. $-4x^2 + 5x = 0$
5. $-x^2 + 2x + 1 = 0$
6. $-\frac{1}{5}x^2 + 2x - 5 = 0$

Réponses :

Ex 1 :

1. $f(x) = -2x^2 + 12x - 14 = -2[(x-3)^2 - 2]$
2. $f(x) = 2x^2 - x + 1 = 2[(x - \frac{1}{4})^2 + \frac{7}{16}]$
3. $f(x) = 2x^2 - x - 1 = 2[(x - \frac{1}{4})^2 - \frac{9}{16}]$
4. $f(x) = 2x^2 - x - 15 = 2[(x - \frac{1}{4})^2 - \frac{121}{16}]$
5. $f(x) = \frac{1}{2}x^2 - x - \frac{3}{2} = \frac{1}{2}[(x-1)^2 - 4]$
6. $f(x) = -\frac{1}{5}x^2 - 2x - 5 = -\frac{1}{5}(x+5)^2$
7. $f(x) = -\frac{1}{3}x^2 + x + 3 = -\frac{1}{3}[(x - \frac{3}{2})^2 - \frac{45}{4}]$
8. $f(x) = 3x^2 - x + \frac{5}{12} = 3[(x - \frac{1}{6})^2 + \frac{1}{9}]$

Ex2 :

1. $4x^2 + 12x + 9 = 0 \iff (2x+3)^2 = 0 \iff 2x + 3 = 0 \iff x = -\frac{3}{2}$ donc $S = \left\{-\frac{3}{2}\right\}$
2. $x^2 - \sqrt{2}x + \frac{1}{2} = 0$ $\Delta = 0$ il y a donc une solution réelle : $x_0 = \frac{\sqrt{2}}{2}$ donc $S = \left\{-\frac{\sqrt{2}}{2}\right\}$
3. $3x^2 - \sqrt{6}x + 1 = 0$ $\Delta = -6 < 0$ il n'y a donc pas de solution réelle : $S = \emptyset$
4. $-4x^2 + 5x = 0 \iff x(-4x+5) = 0 \iff x = 0$ ou $x = \frac{5}{4}$ donc $S = \left\{\frac{5}{4}; 0\right\}$
5. $-x^2 + 2x + 1 = 0$ $\Delta = 8$ il y a donc deux solutions réelles :
   $x_1 = \frac{-2 - 2\sqrt{2}}{-2} = \frac{-2(1+\sqrt{2)}}{-2} = 1 + \sqrt{2}$ et $x_2 = \frac{-2 + 2\sqrt{2}}{-2} = \frac{-2(1-\sqrt{2)}}{-2} = 1 - \sqrt{2}$ donc $S = \left\{1+\sqrt{2}; 1-\sqrt{2}\right\}$
6. $-\frac{1}{5}x^2 + 2x - 5 = 0$ $\Delta = 0$ il y a donc une solution réelle : $x_0 = \frac{-2}{-\frac{2}{5}} = 5$ donc $S = \{5\}$